Measurement of SOA Linewidth Enhancement Factor
With a Sagnac Fiber Loop

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Abstract—We demonstrate a novel method to measure the linewidth enhancement factor (α-factor) of the semiconductor optical amplifier (SOA). We derived theoretically the quantitative relationships among the α-factor, the cross-gain modulation, the cross-phase modulation, and the contrast ratio of an SOA-based Sagnac fiber loop. We found that the α-factor value can be calculated directly from the maximum power contrast ratio measurement. Our experiment results show that the obtained α-factor fluctuates within a small range of 5.23 to 6.83 when the bias current varies from 130 to 240 mA. Compared with existing measurement methods, our method is more attractive because of its simple configuration and better stability.

Index Terms—Linewidth enhancement factor (α-factor), phase modulation, Sagnac fiber loop, semiconductor optical amplifier (SOA).

I. INTRODUCTION

ALL-OPTICAL signal processing concepts have received considerable attention during the past years. Many approaches have been proposed to implement all-optical logic functions, based on the nonlinear effects either in an optical fiber or in a semiconductor optical amplifier (SOA). Compared with using the nonlinearity of optical fibers, all-optical logic gates based on SOA have demonstrated great potential in terms of high speed, small footprint, low power consumption, and ease of optical integration [1]. Linewidth enhancement factor (α-factor), one of the SOA’s main parameters, is of great importance in evaluating the SOA’s performance for cross-phase modulation (XPM)-based applications [2]. The first usage of the α-factor can be traced back to the early 1980s, when first measurements reveal that the semiconductor laser has a linewidth much broader than what was predicted by the Shawlow–Townes theory [3]. The α-factor is discovered to influence several fundamental aspects of semiconductor lasers, such as the linewidth, the chirp under current modulation, the power fluctuation, and mode stability [4]. Since then, a number of theoretical and experimental works were carried out for determining the α-factor in practical devices. The measurement methods include optical self-locking [5], modified direct frequency modulation [6], and injection locking [7]. However, those methods are only suitable to semiconductor lasers.

An SOA is similar to a semiconductor laser without or with negligible optical facet feedback. Therefore, the measurable method of SOA α-factor should be different from that of the semiconductor laser. Theoretically, α-factor of an SOA can be measured by injecting a continuous-wave (CW) optical signal into SOA and superimposing a small signal in sinusoidal modulation (larger than 1 GHz) to the bias current. The measurable values of the amplitude and phase modulation index at the SOA output port are then used to calculate the α-factor. The most commonly used methods adopted either the Michelson interferometer [8] or the Mach–Zehnder interferometer [9] as the detecting technique. However, these interferometric techniques are sensitive to environmental fluctuations, which will make the measurement unstable. Moreover, direct modulation of the laser source with high frequency is also difficult to implement experimentally.

In this letter, we propose a new method for measuring the SOA α-factor with a Sagnac fiber loop configuration. The method needs neither spatial interference nor optical spectrum analysis. Besides the ease of operation, the proposed method guarantees the accuracy of measurement, which is true even under the condition of high bias current and large carrier density dynamics. To the best of our knowledge, it is the first time that a Sagnac fiber loop is applied to SOA α-factor measurement.

II. PRINCIPLE OF MEASUREMENT

The main part of our configuration is an SOA-based Sagnac loop in Fig. 1. It consists of a 2 × 2 coupler with a power splitting ratio of 50:50, two wavelength-division multiplexers (WDM1 and WDM2), a JDSU SOA, a polarization controller to bias the loop polarization, and a 100-m single-mode fiber. The transmission matrix of the coupler is

\[
T = \frac{\sqrt{2}}{2} \begin{bmatrix}
1 & i \\
-i & 1
\end{bmatrix}.
\]
If a signal packet with electric field $E_3(t) = f(t)e^{i\omega_0 t}$ at an angular frequency $\omega_0$ is imported into the $2 \times 2$ coupler, it will be split into two output signals with equal power at Ports 3 and 4. The electric fields of these two output signals are

$$
\begin{bmatrix}
  E_3 \\
  E_4
\end{bmatrix}
= \mathbf{T}
\begin{bmatrix}
  E_1 \\
  E_2
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  i
\end{bmatrix}
\frac{\sqrt{2}}{2} f(t) \exp(i\omega_0 t),
$$

(2)

Then $E_3$ and $E_4$ will propagate in clockwise (CW) and counter-clockwise (CCW) directions in the fiber loop, respectively. The CW-traveling signal will be amplified by the complex gain of SOA as $g_{cw}(t) = g_{cw}(t)e^{-i\phi_{cw}}$. Likewise, the CCW-traveling signal will experience a gain as $g_{ccw}(t) = g_{ccw}(t)e^{-i\phi_{ccw}}$. Here $g_{cw}(t)$ and $g_{ccw}(t)$ are the SOA amplitude gains, and $\phi_{cw}(t)$ and $\phi_{ccw}(t)$ are the phase shifts for signals propagating in CW and CCW directions, respectively. If no control signal exists in the fiber loop, two counterpropagating signals experience the same phase shift $\phi = \phi_{cw} = \phi_{ccw}$. After propagation for one loop, the two signals will interfere at the coupler on the condition that they have the same polarization state, and the interference result is calculated as

$$
\begin{bmatrix}
  E_1 \\
  E_2
\end{bmatrix}
= \mathbf{T}
\begin{bmatrix}
  E_3 \\
  E_4
\end{bmatrix}
= \begin{bmatrix}
  \frac{1}{2} e^{-\sigma L/2} e^{i\phi(t)}[g_{cw}(t - \tau) + g_{ccw}(t - \tau)] \\
  \frac{1}{2} e^{-\sigma L/2} e^{i\phi(t)}[g_{cw}(t - \tau) - g_{ccw}(t - \tau)]
\end{bmatrix}
\begin{bmatrix}
  1 \\
  e^{i\phi_{cw}}
\end{bmatrix}
\begin{bmatrix}
  \frac{1}{2} e^{-\sigma L/2} e^{i\phi(t)}\sin(t - \tau) \\
  0
\end{bmatrix}
\begin{bmatrix}
  e^{i\phi_{cw}}
\end{bmatrix}
= \begin{bmatrix}
  \frac{1}{4} e^{-\sigma L} P_{in} g_{cw} \sin(t - \tau) + 2 \sqrt{\frac{G_{cw}}{G_{ccw}}} \cos(\Delta \varphi) \\
  \frac{1}{4} e^{-\sigma L} P_{in} g_{ccw} \sin(t - \tau) - 2 \sqrt{\frac{G_{cw}}{G_{ccw}}} \cos(\Delta \varphi)
\end{bmatrix}
$$

(3)

where $\sigma/2$, $L$, and $\tau$ are electrical field attenuation coefficient, fiber length in the Sagnac loop, and the differential group delay of the fiber loop, respectively. If we assume the system is ideal, i.e., $g_{cw}(t) = g_{ccw}(t) = g$, the injected packet signal will be totally directed to Port C3 of the circulator. Now if we introduce another signal with wavelength $\lambda_{pump}$ synchronized with the CW signal into SOA, a gain difference and phase difference $\Delta \phi = \phi_{cw} - \phi_{ccw}$ will be present. If we omit the common phase shift $\phi = \phi_1 + \phi_2/2$, the interference result can be derived as

$$
\begin{bmatrix}
  P_1 \\
  P_2
\end{bmatrix}
= \frac{1}{4} P_{in} g_{cw} \sin(t - \tau) \left[
\begin{array}{c}
  1 + \frac{G_{cw}}{G_{ccw}} + 2 \sqrt{\frac{G_{cw}}{G_{ccw}}} \cos(\Delta \varphi) \\
  1 + \frac{G_{ccw}}{G_{cw}} - 2 \sqrt{\frac{G_{cw}}{G_{ccw}}} \cos(\Delta \varphi)
\end{array}
\right]
$$

(4)

where $P_{in} = f^2(t - \tau)$ is the input signal power, and $G_{cw} = g_{cw}^2$, $G_{ccw} = g_{ccw}^2$, are the power gain in CW and CCW directions, respectively. The output power depends on the nonlinear phase shift $\Delta \phi$ and gain ratio $m = \sqrt{G_{cw}/G_{ccw}}$ for two different states of SOA saturation, which are coupled by [10]

$$
\Delta \phi = -\frac{\alpha}{2} \ln \left( \frac{g_{cw}}{g_{ccw}} \right) = -\frac{\alpha}{2} \ln (m).
$$

(5)

In (5), $\alpha$ is the linewidth enhancement factor. If we define the power contrast ratio (PCR) as $P_2/P_1$, based on (4) and (5), one $\alpha$-factor measurement method can be deduced. That is, we can first measure $G_{cw}$ and $G_{ccw}$ values, and then calculate the nonlinear phase shift $\Delta \varphi$ from PCR using (4), followed by determining the $\alpha$-factor using (5). We refer to this procedure as approach-1. This approach is not easy to implement. In fact, the derivation procedure can be simplified. If we combine (4) and (5), we can get

$$
\begin{bmatrix}
  P_1 \\
  P_2
\end{bmatrix}
= \frac{1}{4} e^{-\sigma L} P_{in} G_{cw}
\begin{bmatrix}
  \left[ 1 + e^{-4(\Delta \varphi/\alpha)} + 2 e^{-(2\lambda \varphi/\alpha) \cos(\Delta \varphi)} \right] \\
  \left[ 1 + e^{-4(\Delta \varphi/\alpha)} - 2 e^{-(2\lambda \varphi/\alpha) \cos(\Delta \varphi)} \right]
\end{bmatrix}
$$

(6)

According to (6), it is possible to plot the variation of PCR with respect to the nonlinear phase shift as in Fig. 2. We can observe that for the $\alpha$-factor greater than or equal to four, the PCR will achieve a maximum value around $\Delta \varphi = 0.9\pi$. To simplify the final formula, we assume $\cos(0.9\pi) = -1$. Therefore, we obtain a simple formula for calculating the $\alpha$-factor from the maximum PCR

$$
\left( \frac{P_2}{P_1} \right)_{\text{max}} = \left( 1 + \exp \left( -\frac{1.8\pi}{\alpha} \right) \right)^2.
$$

(7)

We refer to this approach as approach-2. To validate the second approach, we have carried out an error analysis in the inset of Fig. 2. It is obvious that the measurement result has only less than 5% error as compared to approach-1.

### III. Experiment Results

In our measurement, two DFB laser modules with maximum output power of 10 dBm at different wavelengths are used as the control ($\lambda = 1556.56$ nm) and packet signal ($\lambda = 1559.89$ nm) light sources, respectively. Pulse patterns of the control and packet signals are generated with a LiNbO$_3$ modulator and an acoustic-optical modulator driven by a pulse pattern generator and a function waveform generator, respectively. We edit the 100-ns packet signal with a 10-µs cycle, and also generate the control signal with a 20-µs cycle to synchronize with the packet signal at point A in Fig. 1. From our theoretical analysis, our method does not depend on the bit rate of the input signal; therefore, it is appropriate to generate a 155-Mb/s packet signal for
we fix the input power as $-3$ dBm. As the bias current grows from 130 to 240 mA, the $\alpha$-factor only fluctuates within a small range of 5.23–6.83. The increase in $\alpha$-factor is directly caused by the increase of free carrier density. The inset of Fig. 4 depicts the $\alpha$-factor for different input signal powers at a constant bias current (200 mA). The experimental measurement indicates that the $\alpha$-factor slightly decreases with increasing input power. In this case, the decrease is mainly due to the SOA saturation from input signal. The experimental results are in good agreement with the product datasheet (it is specified as a constant 6) as well as results presented in [2]. Therefore, we believe that those two approaches are accurate enough to be applied in real measurements.

IV. CONCLUSION

A novel method for measuring SOA $\alpha$-factor is proposed. The new methodology makes use of an SOA-based Sagnac fiber loop, and the theoretical formulas are derived according to the co-existence of XPM, cross-gain modulation, and the contrast ratio of the SOA-based Sagnac fiber loop. Our theoretical analysis provides two possible approaches for $\alpha$-factor calculation. Approach-2 is to calculate the $\alpha$-factor directly from the maximum PCR. Approach-1 is more general compared to approach-2, since it does not need to satisfy the specific nonlinear phase shift condition. The tradeoff of approach-1 is that the SOA gains in CW and CCW directions need to be measured in advance and more tedious calculations are involved. The validity of both approaches is confirmed with the experimental results. Considering the fact that a Sagnac fiber loop is more stable than other interferometric setups, we believe that the accuracy and the stability of our method are better.

REFERENCES