Virtual Generalized Mueller Matrix Method for Measurement of Complex Polarization-Mode Dispersion Vector in Optical Fibers

H. Dong, P. Shum, Y. D. Gong, M. Yan, J. Q. Zhou, and C. Q. Wu

***Abstract*—A virtual generalized Mueller matrix method (VGMMM) is proposed to measure the complex polarization-mode dispersion (PMD) vector in a fiber system with polarization-dependent loss or gain. VGMMM can attain the low-noise high-resolution PMD data using a relatively large frequency step, without the knowledge of input polarization states. VGMMM combines the advantages of both matrix-based methods and differentiation-based methods and overcomes their shortcomings. Experimental results on a fiber system confirm the validity and accuracy of VGMMM.**

***Index Terms*—Complex polarization-mode dispersion (PMD) vector, Mueller matrix, polarization-dependent loss or gain (PDL/G), polarization-mode dispersion (PMD).**

I. Introduction

POLARIZATION-MODE dispersion (PMD) is completely described by the complex PMD vector $\vec{W}(\omega) = \vec{\Omega}(\omega) + i\vec{\Lambda}(\omega)$ in optical fibers with polarization-dependent loss or gain (PDL/G) [1]. Up to now, measurement methods for the complex PMD vector in frequency domain are divided into two categories. The first category is based on the differential equations, which involves derivatives of polarization states with respect to optical frequency. Such methods include the complex plane method [2] and the generalized Poincaré sphere method (GPST) [3]. The second category is based on analysis of the transmission matrices, including Jones matrix eigenanalysis (JME) method [4] and generalized Mueller matrix method (GMMM) [5]. The measurement methods in the first category do not require the knowledge of input polarization states, so only a polarization controller (PC) is needed for changing the input polarization states [2]. However, these measurement methods suffer from high noise due to high-precision demand in setting up the frequencies with a very small step size as required by differentiation [5]. In the second category, the existing JME method does not present all information of the complex PMD vector [4]. Although the GMMM has been demonstrated to be a good technique to attain low-noise high-resolution PMD data [5], it, unfortunately, requires the knowledge of the input polarization states. Therefore, a rotatable polarizer or a polarization-state generator has to be inserted into the GMMM measurement system [5]. Not only that the system configuration is more complex, but also setup of the predefined input polarization states tends to introduce errors. A newly presented method which belongs to the second category makes use of the least-square analysis of Mueller matrix [6], [7]. This technique can determine the first- and higher order PMD effects simultaneously by employing decades of unknown input polarization states. It is a very good idea for theoretical investigation; but for experimental realization, its accuracy is affected by the limited repeatability of the tunable laser source and polarization state generator as well as the instability of the fiber system under test. In this letter, a virtual GMMM (VGMMM) is proposed. Compared to previous methods, this technique can measure the complex PMD vector using a relatively large frequency step without the knowledge of three input polarization states. Measurement accuracy can be greatly improved by optimizing the frequency step using a theoretical relation. Experimentally, system setup is simplified as the knowledge of input states is not required. Experimental results on a fiber system with bending-induced PDL confirm the validity and accuracy of the proposed VGMMM.

Manuscript received June 15, 2006; revised October 11, 2006. This work was supported in part by the Agency for Science, Technology and Research (A $*$ Star), Singapore, under Project 042 101 0015, and by the Open Fund of Key Laboratory of Optical Communication and Lightwave Technologies, Beijing University of Posts and Telecommunications, Ministry of Education, China.

H. Dong, P. Shum, M. Yan, and J. Q. Zhou are with the Network Technology Research Centre, Nanyang Technological University, Singapore 637553, Singapore.

Y. D. Gong is with the Institute for InfoComm Research, Singapore 119613, Singapore.

C. Q. Wu is with the School of Science, Beijing Jiaotong University, Beijing 100044, China.

Digital Object Identifier 10.1109/LPT.2006.887888

II. Theory

For a general optical fiber system with PDL/G, if the input polarization state $\vec{S}_{\rm in} = (\, s_{\rm in0} \quad s_{\rm in1} \quad s_{\rm in2} \quad s_{\rm in3}\,)^T$ is fixed and the optical frequency is being swept, at the output end of the system, the output polarization states at two adjacent frequencies are related by [5]

$$\vec{S}_{\rm out}(\omega + \Delta\omega) = M(\omega + \Delta\omega)M^{-1}(\omega)\vec{S}_{\rm out}(\omega). \quad (1)$$

Here M denotes a 4×4 Mueller matrix. The matrix $M_{\Delta}(\omega) = M(\omega + \Delta\omega)M^{-1}(\omega)$ contains the information about the complex PMD vector. Based on the polar decomposition of $M_{\Delta}(\omega)$, the complex PMD vector $\vec{W}(\omega) = \vec{\Omega}(\omega) + i\vec{\Lambda}(\omega)$ can be determined easily with a relatively large $\Delta\omega$ [5]. But in order to obtain $M_{\Delta}(\omega)$, we have to measure the Mueller matrices at two adjacent frequencies. This is the step that requires knowledge of input polarization states [5]. From another point of view, based on (1), $M_{\Delta}(\omega)$ can indeed be considered as the transmission matrix of a virtual optical system in the frequency domain, with $\vec{S}_{\rm out}(\omega)$ as the input and $\vec{S}_{\rm out}(\omega + \Delta\omega)$ as the output. This implies that $M_{\Delta}(\omega)$ can be determined by $\vec{S}_{\rm out}(\omega)$ and $\vec{S}_{\rm out}(\omega + \Delta\omega)$ directly as long as $\vec{S}_{\rm in}$ is fixed, and the knowledge of $\vec{S}_{\rm in}$ is not required at all.

Because $M_\Delta(\omega)$ corresponds to a Lorentz transformation [5], it can be determined using three inputs. For three fixed but unknown input polarization states $\vec{S}_{\rm in}$, $\vec{T}_{\rm in}$, and $\vec{U}_{\rm in}$, their outputs are measured at two adjacent frequencies as $\vec{S}_{\rm out}(\omega)$, $\vec{S}_{\rm out}(\omega+\Delta\omega)$, $\vec{T}_{\rm out}(\omega)$, $\vec{T}_{\rm out}(\omega+\Delta\omega)$, $\vec{U}_{\rm out}(\omega)$, and $\vec{U}_{\rm out}(\omega+\Delta\omega)$. Certainly we can use these outputs and the nine bilinear equations relating the matrix elements of a Lorentz transformation [8] to calculate $M_\Delta(\omega)$, but such procedure will induce a nonlinear equation group, which is difficult to be solved. Alternatively, we can first assume some other four polarization states as $\vec{V}_{\rm out1}(\omega) = (\,1 \quad 1 \quad 0 \quad 0\,)^T$, $\vec{V}_{\rm out2}(\omega) = (\,1 \quad -1 \quad 0 \quad 0\,)^T$, $\vec{V}_{\rm out3}(\omega) = (\,1 \quad 0 \quad 1 \quad 0\,)^T$, and $\vec{V}_{\rm out4}(\omega) = (\,1 \quad 0 \quad 0 \quad 1\,)^T$ at frequency ω. If we can calculate the corresponding outputs $\vec{V}_{{\rm out}j}(\omega+\Delta\omega)$, $j = 1,2,3,4$ at frequency $\omega+\Delta\omega$ from the above-mentioned measurements, $M_\Delta(\omega)$ can be easily determined. We have [8]

$$M_\Delta^T G M_\Delta = \sqrt{\det M_\Delta}\, G. \tag{2}$$

Here $G = \mathrm{diag}(\,1 \quad -1 \quad -1 \quad -1\,)$. Based on (2), we can obtain (3) , shown at the bottom of the page. Here $\vec{S} \otimes \vec{T} = s_0t_0 - s_1t_1 - s_2t_2 - s_3t_3$, $\sqrt{\det M_\Delta} = \vec{S}_{\rm out}(\omega+\Delta\omega) \otimes \vec{T}_{\rm out}(\omega+\Delta\omega) / \vec{S}_{\rm out}(\omega) \otimes \vec{T}_{\rm out}(\omega)$, and $Vol_{4D} = \det \begin{pmatrix} s_0 & s_1 & s_2 & s_3 \\ t_0 & t_1 & t_2 & t_3 \\ u_0 & u_1 & u_2 & u_3 \\ v_0 & v_1 & v_2 & v_3 \end{pmatrix}$. Equation (3) is a linear equation group, $\vec{V}_{{\rm out}j}(\omega+\Delta\omega)$ can be calculated as long as $\vec{S}_{\rm in}$, $\vec{T}_{\rm in}$, and $\vec{U}_{\rm in}$ are linearly independent. It should be noted that these vectors are four-dimensional vectors. In three-dimensional (3-D) Stokes space, the three inputs need not to be linearly independent. A well-known case is to use $(\,1 \quad 0 \quad 0\,)^T$, $(\,-1 \quad 0 \quad 0\,)^T$, and $(\,0 \quad 1 \quad 0\,)^T$ as inputs, which is employed in JME [4] and GMMM [5]. In fact, we can prove that the condition for (3) being solvable is the three 3-D inputs are not linearly superposed with each other in Stokes space. This can be easily guaranteed in the experimental setup. Moreover, we can also prove the optimum accuracy is achievable when the three 3-D inputs are coplanar and the angles between them are 120° in Stokes space [e.g., $(\,1 \quad 0 \quad 0\,)^T$, $(\,-1/2 \quad \sqrt{3}/2 \quad 0\,)^T$, and $(\,-1/2 \quad -\sqrt{3}/2 \quad 0\,)^T$]. Eventually, $M_\Delta(\omega)$ can be determined as (4) , shown at the bottom of the page.

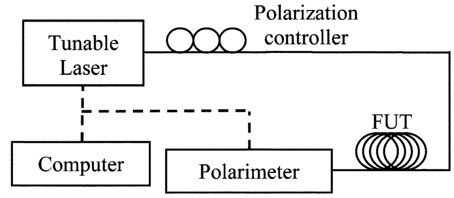


Fig. 1. Experimental setup.

The complex PMD vector can be extracted from $M_\Delta(\omega)$ according to [5, eq. (6), (7), (8), and (10)]; and the differential group delay (DGD) and differential attenuation slope can be obtained from [5, eq. (11)]. The technique in [5] works fine if the higher order PMD effects are negligible at the two adjacent frequencies, which means $\mathrm{DGD}\cdot\Delta\omega$ should be small enough. But a too small $\Delta\omega$ may cause inaccuracies [5]. So there is a tradeoff for the choice of $\Delta\omega$. We will clearly show this in Section III.

III. Experimental Results

To verify VGMMM, an experimental setup is established as shown in Fig. 1. The manually controlled PC is used to generate three fixed but unknown inputs. Although they are unknown, they are close to be coplanar with 120° angles by tuning PC to three precalibrated states; so the optimum accuracy can be approached to a certain extent. The fiber under test (FUT) consists of two 1-km-long highly nonlinear fibers, with a coiled fiber section (coil diameter at 1.5 cm) in between to induce PDL [9]. We make this measurement in the wavelength range from 1525 to 1555 nm with a 0.1-nm step size.

To choose an optimum wavelength step size, we compare the experimental results under different wavelength step sizes with the theoretical relation $\langle\vec{\Omega}\cdot\vec{\Lambda}\rangle = 0$ [1]. To do this, we calculate $\alpha = |\langle\vec{\Omega}\cdot\vec{\Lambda}\rangle| / \sqrt{\langle\Omega^2\rangle\langle\Lambda^2\rangle}$ with different step sizes and show them in Fig. 2. From Fig. 2, although theoretically $\Delta\lambda = 0.1$ nm should have the smallest impact of the higher order effects, it cannot achieve the best accuracy due to errors induced by the instruments. $\Delta\lambda = 0.5$ nm achieves the best accuracy, but some real data is possible to be overlooked when wavelength step size is too large. $\Delta\lambda = 0.2$ nm gives a good accuracy and is not too large; therefore, it is the optimum wavelength step size in our experiment. To compare the measurement results of VGMMM with those of JME, we employ the measurement

$$\begin{cases} \vec{S}_{\rm out}(\omega+\Delta\omega) \otimes \vec{V}_{{\rm out}j}(\omega+\Delta\omega) = \sqrt{\det M_\Delta}\,\vec{S}_{\rm out}(\omega) \otimes \vec{V}_{{\rm out}j}(\omega) \\ \vec{T}_{\rm out}(\omega+\Delta\omega) \otimes \vec{V}_{{\rm out}j}(\omega+\Delta\omega) = \sqrt{\det M_\Delta}\,\vec{T}_{\rm out}(\omega) \otimes \vec{V}_{{\rm out}j}(\omega) \\ \vec{U}_{\rm out}(\omega+\Delta\omega) \otimes \vec{V}_{{\rm out}j}(\omega+\Delta\omega) = \sqrt{\det M_\Delta}\,\vec{U}_{\rm out}(\omega) \otimes \vec{V}_{{\rm out}j}(\omega) \\ Vol_{4D}^{\omega+\Delta\omega} = \det M_\Delta \cdot Vol_{4D}^{\omega} \end{cases} \tag{3}$$

$$M_\Delta(\omega) = \frac{\begin{pmatrix} \vec{V}_{\rm out1}^T(\omega+\Delta\omega) + \vec{V}_{\rm out2}^T(\omega+\Delta\omega) \\ \vec{V}_{\rm out1}^T(\omega+\Delta\omega) - \vec{V}_{\rm out2}^T(\omega+\Delta\omega) \\ 2\vec{V}_{\rm out3}^T(\omega+\Delta\omega) - \vec{V}_{\rm out1}^T(\omega+\Delta\omega) - \vec{V}_{\rm out2}^T(\omega+\Delta\omega) \\ 2\vec{V}_{\rm out4}^T(\omega+\Delta\omega) - \vec{V}_{\rm out1}^T(\omega+\Delta\omega) - \vec{V}_{\rm out2}^T(\omega+\Delta\omega) \end{pmatrix}}{2} \tag{4}$$

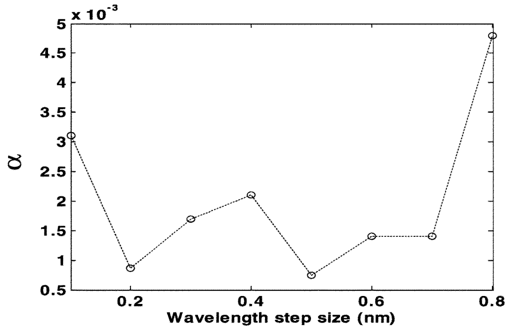


Fig. 2. α values measured with different wavelength step sizes.

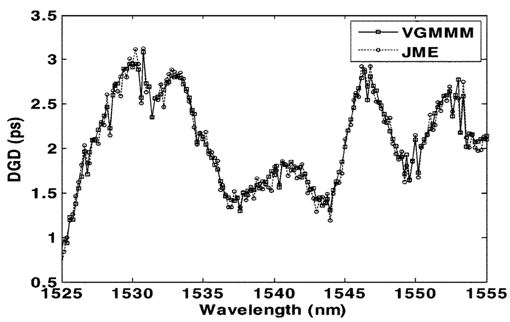


Fig. 3. Measured DGD using VGMMM and JME with $\Delta\lambda = 0.2$ nm.

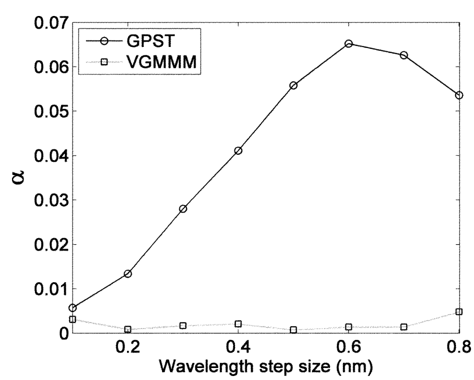
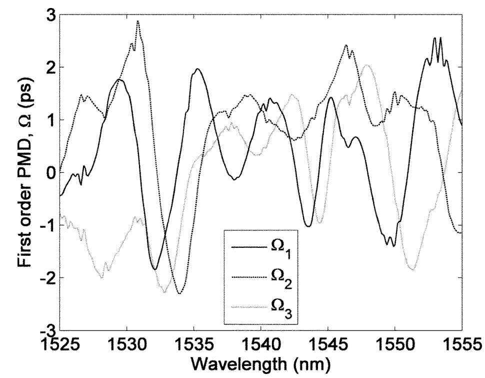
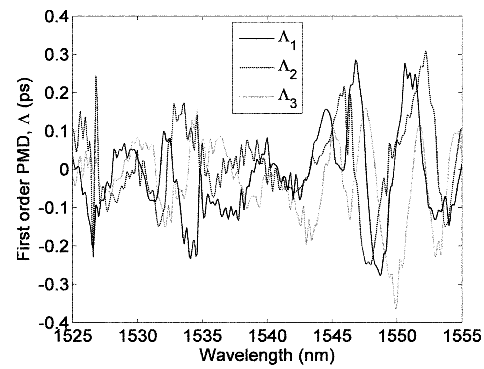


Fig. 4. α values measured using VGMMM and GPST.

setup with a polarization state generator shown in [5]. DGD values are measured using two methods with $\Delta\lambda = 0.2$ nm and the results are plotted in Fig. 3. The good agreement between two methods is clear. Moreover, we compare α values measured using VGMMM and GPST, as shown in Fig. 4. It is shown that VGMMM is obviously more accurate than GPST. Finally, the measured $\vec{\Omega}$ and $\vec{\Lambda}$ using our proposed VGMMM with $\Delta\lambda = 0.2$ nm are shown in Fig. 5.



(a)



(b)

Fig. 5. Measured first-order complex PMD vector (a) $\vec{\Omega}$ and (b) $\vec{\Lambda}$ using VGMMM.

IV. Conclusion

VGMMM is proposed to measure the complex PMD vector in optical fiber systems with PDL/G. This technique can use a relatively large frequency step to attain low-noise high-resolution PMD vector data without the knowledge of input polarization states. VGMMM combines the advantages of both matrix-based methods and differentiation-based methods and overcomes their shortcomings. By comparing the measured data and theoretical relation, the optimum wavelength step size can be determined. Experimentally, VGMMM is shown to agree with JME, and the accuracy of VGMMM is confirmed by comparing with theoretical relation and GPST method.

References

[1] Y. Li and A. Yariv, "Solutions to the dynamical equation of polarization-mode dispersion and polarization-dependent losses," *J. Opt. Soc. Amer. B*, vol. 17, pp. 1821–1827, 2000.

[2] A. Eyal and M. Tur, "Measurement of polarization-mode dispersion in systems having polarization-dependent loss or gain," *IEEE Photon. Technol. Lett.*, vol. 9, no. 9, pp. 1256–1258, Sep. 1997.

[3] L. Chen, O. Chen, S. Hadjifaradji, and X. Bao, "Polarization-mode dispersion measurement in a system with polarization-dependent loss or gain," *IEEE Photon. Technol. Lett.*, vol. 16, no. 1, pp. 206–208, Jan. 2004.

[4] B. L. Heffner, "Automated measurement of polarization mode dispersion using Jones matrix eigenanalysis," *IEEE Photon. Technol. Lett.*, vol. 4, no. 9, pp. 1066–1069, Sep. 1992.

[5] H. Dong, P. Shum, M. Yan, G. Ning, Y. Gong, and C. Wu, "Generalized Mueller matrix method for polarization mode dispersion measurement in a system with polarization-dependent loss or gain," *Opt. Express*, vol. 14, pp. 5067–5072, 2006.

[6] M. Reimer and D. Yevick, "Least squares procedure for measuring Mueller matrices," in *Proc. OFC*, Anahaeim, CA, 2006, Paper OWI41.

[7] ——, "Least-squares analysis of Mueller matrix," *Opt. Lett.*, vol. 31, pp. 2399–2401, 2006.

[8] R. Barakat, "Bilinear constraints between elements of the 4×4 Mueller-Jones transfer matrix of polarization theory," *Opt. Commun.*, vol. 38, pp. 159–161, 1981.

[9] A. B. dos Santos and J. P. von der weid, "PDL effects in PMD emulators made out with HiBi fibers: Building PMD/PDL emulators," *IEEE Photon. Technol. Lett.*, vol. 16, no. 2, pp. 452–454, Feb. 2004.