

Spectrally resolved reflectometric measurement of polarization mode dispersion in an optical fiber link with polarization-dependent loss

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In an optical fiber link with polarization-dependent loss (PDL), we demonstrate that although the complex polarization mode dispersion vector cannot be fully obtained by the reflectometric measurement, the spectrally resolved differential group delay (DGD) and differential attenuation slope (DAS) can be explicitly determined by such measurements performed simultaneously in the optical frequency domain and the fiber length domain. In principle, this technique can be used to realize the spectrally resolved and spatially resolved measurement of DGD and DAS in an optical fiber link having PDL based on distributed Rayleigh backscattering. We report the experimental result based on the far-end Fresnel reflection to confirm the validity of the proposed method. © 2007 Optical Society of America
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The reflectometric technique has been proposed to perform single-end and spatially resolved polarization mode dispersion (PMD) measurement in optical fiber links with the advantages of single-end access and nondestructive distributed measurement [1–3]. Until now, this technique could only be used to measure an optical fiber link without polarization-dependent loss (PDL). In an optical fiber link with PDL, because of the round-trip effect, the round-trip complex PMD vector $\vec{W}_B = \vec{\Omega}_B + i\vec{\Lambda}_B$, which can be measured directly in the test, is explicitly related to only the first two elements of the complex PMD vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$ as [4]

$$\Omega_B^2 - \Lambda_B^2 = 4(\Omega_L^2 - \Lambda_L^2), \quad \vec{\Omega}_B \cdot \vec{\Lambda}_B = 4\vec{\Omega}_L \cdot \vec{\Lambda}_L, \quad (1)$$

where $\vec{\Omega}_L = (\Omega_1, \Omega_2, 0)^T$ and $\vec{\Lambda}_L = (\Lambda_1, \Lambda_2, 0)^T$ are the linear parts of the complex PMD vector and T denotes the matrix transpose. Because there are too few equations, Eq. (1) cannot be uniquely solved [4]. Even if it could be uniquely solved under some assumptions, spectrally resolved PMD measurement still could not be achieved owing to the lack of information about the third elements of the complex PMD vector, Ω_3 and Λ_3 . In this Letter, we demonstrate theoretically and experimentally that spectrally resolved reflectometric PMD measurement in optical fibers with PDL can be accomplished. Based on the PMD dynamic equation, two equations can be obtained to relate $\Omega_3\Lambda_3$, $\Omega_3^2 - \Lambda_3^2$ to the round-trip complex PMD vector and round-trip local birefringence vector, which can be measured by reflectometric measurements carried out simultaneously in the optical frequency domain and the fiber length domain. Then, by combining these equations with Eqs. (1) and (10), the spectrally resolved differential group delay (DGD) and differential attenuation slope (DAS) can be deterministically achieved, although the principal

states of polarization still cannot be determined. Optical fibers used in telecommunication systems always have negligible PDL values unless a fiber is bent with a small diameter. The PDL in an optical fiber system is caused mainly by some optical components, such as couplers and filter. For one optical component, a typical PDL value is around 0.2 dB, and the total PDL value of an optical fiber system with many PDL components may be large. From the previously reported measurement results using the forward measurement technique, we know that even a PDL of 0.1 dB can have an obvious impact on the PMD measurement [5]. Therefore the proposed technique has to be used even if the PDL of the fiber link under test is not very large.

For a fiber system with both birefringence and PDL, its Mueller matrix \mathbf{M} is amenable to the Lorentz transformation [6]. Thus, it has been demonstrated theoretically and experimentally that if we normalize \mathbf{M} to make $\det \mathbf{M} = 1$, then [7]

$$\mathbf{B} = \frac{d\mathbf{M}}{dz} \mathbf{M}^{-1} = \begin{bmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_1 & 0 & -\beta_3 & \beta_2 \\ \alpha_2 & \beta_3 & 0 & -\beta_1 \\ \alpha_3 & -\beta_2 & \beta_1 & 0 \end{bmatrix},$$

$$\mathbf{P} = \frac{d\mathbf{M}}{d\omega} \mathbf{M}^{-1} = \begin{bmatrix} 0 & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1 & 0 & -\Omega_3 & \Omega_2 \\ \Lambda_2 & \Omega_3 & 0 & -\Omega_1 \\ \Lambda_3 & -\Omega_2 & \Omega_1 & 0 \end{bmatrix}, \quad (2)$$

where $\vec{\alpha} = (\alpha_1 \ \alpha_2 \ \alpha_3)^T$ and $\vec{\beta} = (\beta_1 \ \beta_2 \ \beta_3)^T$ are the local PDL vector and birefringence vector, respectively; $\vec{\Omega} = (\Omega_1 \ \Omega_2 \ \Omega_3)^T$ and $\vec{\Lambda} = (\Lambda_1 \ \Lambda_2 \ \Lambda_3)^T$ are the real and imaginary parts of the complex PMD vector. In reflec-

tometric measurement, the round-trip Mueller matrix should be $\mathbf{M}_B = \mathbf{R}\mathbf{M}^T\mathbf{R}\mathbf{M}$ with $\mathbf{R} = \text{diag}(1 \ 1 \ 1 \ -1)$ [4], and it is also a Lorentz transformation. Then we have

$$\mathbf{B}_B = \frac{d\mathbf{M}_B}{dz}\mathbf{M}_B^{-1} = \begin{bmatrix} 0 & \alpha_{B1} & \alpha_{B2} & \alpha_{B3} \\ \alpha_{B1} & 0 & -\beta_{B3} & \beta_{B2} \\ \alpha_{B2} & \beta_{B3} & 0 & -\beta_{B1} \\ \alpha_{B3} & -\beta_{B2} & \beta_{B1} & 0 \end{bmatrix},$$

$$\mathbf{P}_B = \frac{d\mathbf{M}_B}{d\omega}\mathbf{M}_B^{-1} = \begin{bmatrix} 0 & \Lambda_{B1} & \Lambda_{B2} & \Lambda_{B3} \\ \Lambda_{B1} & 0 & -\Omega_{B3} & \Omega_{B2} \\ \Lambda_{B2} & \Omega_{B3} & 0 & -\Omega_{B1} \\ \Lambda_{B3} & -\Omega_{B2} & \Omega_{B1} & 0 \end{bmatrix}, \quad (3)$$

where $\vec{a}_B = (\alpha_{B1} \ \alpha_{B2} \ \alpha_{B3})^T$ and $\vec{\beta}_B = (\beta_{B1} \ \beta_{B2} \ \beta_{B3})^T$ are the local round-trip PDL vector and birefringence vector, respectively; $\vec{\Omega}_B = (\Omega_{B1} \ \Omega_{B2} \ \Omega_{B3})^T$ and $\vec{\Lambda}_B = (\Lambda_{B1} \ \Lambda_{B2} \ \Lambda_{B3})^T$ are the real and imaginary parts of the round-trip complex PMD vector. The differentiations of Eqs. (3) are

$$\frac{\partial \mathbf{P}_B}{\partial z} = 2\mathbf{R}\mathbf{M}^T\mathbf{R} \left(\mathbf{R}\mathbf{B}^T\mathbf{R}\mathbf{P}_L - \mathbf{P}_L\mathbf{R}\mathbf{B}^T\mathbf{R} + \frac{\partial \mathbf{P}_L}{\partial z} \right) \times (\mathbf{R}\mathbf{M}^T\mathbf{R})^{-1},$$

$$\frac{\partial \mathbf{B}_B}{\partial \omega} = 2\mathbf{R}\mathbf{M}^T\mathbf{R} \left(\mathbf{R}\mathbf{P}^T\mathbf{R}\mathbf{B}_L - \mathbf{B}_L\mathbf{R}\mathbf{P}^T\mathbf{R} + \frac{\partial \mathbf{B}_L}{\partial \omega} \right) \times (\mathbf{R}\mathbf{M}^T\mathbf{R})^{-1}, \quad (4)$$

where \mathbf{B}_L and \mathbf{P}_L are the simple cases of matrices \mathbf{B} and \mathbf{P} defined in Eq. (2) when $\vec{a} = (\alpha_1 \ \alpha_2 \ 0)^T$, $\vec{\beta} = (-\beta_1 \ \beta_2 \ 0)^T$ and $\vec{\Omega} = (-\Omega_1 \ \Omega_2 \ 0)^T$, $\vec{\Lambda} = (-\Lambda_1 \ \Lambda_2 \ 0)^T$. Taking the sum of Eqs. (4), we obtain

$$\frac{\partial \mathbf{P}_B}{\partial z} + \frac{\partial \mathbf{B}_B}{\partial \omega} = 2\mathbf{R}\mathbf{M}^T\mathbf{R} \begin{bmatrix} 0 & C_1 & C_2 & 0 \\ C_1 & 0 & 0 & D_2 \\ C_2 & 0 & 0 & -D_1 \\ 0 & -D_2 & D_1 & 0 \end{bmatrix} \times (\mathbf{R}\mathbf{M}^T\mathbf{R})^{-1}, \quad (5)$$

where $C_1 = \beta_3\Lambda_2 + \alpha_3\Omega_2 + \alpha_2\Omega_3 + \beta_2\Lambda_3 + \partial\alpha_1/\partial\omega + \partial\Lambda_1/\partial z$, $C_2 = -\beta_3\Lambda_1 - \alpha_3\Omega_1 - \alpha_1\Omega_3 - \beta_1\Lambda_3 + \partial\alpha_2/\partial\omega + \partial\Lambda_2/\partial z$, $D_1 = \beta_3\Omega_2 - \alpha_3\Lambda_2 - \alpha_2\Lambda_3 + \beta_2\Omega_3 + \partial\beta_1/\partial\omega + \partial\Omega_1/\partial z$, $D_2 = -\beta_3\Omega_1 + \alpha_3\Lambda_1 + \alpha_1\Lambda_3 - \beta_1\Omega_3 + \partial\beta_2/\partial\omega + \partial\Omega_2/\partial z$. On the other hand, we can easily obtain the PMD dynamical equation from Eqs. (2) as

$$\frac{\partial \vec{\Omega}}{\partial z} = \frac{\partial \vec{\beta}}{\partial \omega} + \vec{\beta} \times \vec{\Omega} - \vec{a} \times \vec{\Lambda},$$

$$\frac{\partial \vec{\Lambda}}{\partial z} = \frac{\partial \vec{\alpha}}{\partial \omega} + \vec{\beta} \times \vec{\Lambda} + \vec{a} \times \vec{\Omega}. \quad (6)$$

In optical fibers and components used in optical communications, at least over the wavelength range of interest, we have [8,9]

$$\frac{\partial \vec{\alpha}}{\partial \omega} = 0, \quad \frac{\partial \vec{\beta}}{\partial \omega} = \frac{\vec{\beta}}{\omega}. \quad (7)$$

Applying Eqs. (6) and (7), the elements in Eq. (5) can be simplified as $C_1 = 2(\alpha_2\Omega_3 + \beta_2\Lambda_3)$, $C_2 = -2(\alpha_1\Omega_3 + \beta_1\Lambda_3)$, $D_1 = 2[(\beta_1/\omega) + \beta_2\Omega_3 - \alpha_2\Lambda_3]$, $D_2 = 2[(\beta_2/\omega) + \alpha_1\Lambda_3 - \beta_1\Omega_3]$. Because the right- and left-hand sides of Eq. (5) are similar matrices, they should have the same eigenvalues. Then two equations can be obtained as

$$16(\vec{\alpha}_L \cdot \vec{\beta}_L)(\Omega_3^2 - \Lambda_3^2) + 16(\beta_L^2 - \alpha_L^2)\Omega_3\Lambda_3$$

$$+ 16\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Omega_3 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right) \cdot \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega} \right),$$

$$16(\beta_L^2 - \alpha_L^2)(\Omega_3^2 - \Lambda_3^2) - 64(\vec{\alpha}_L \cdot \vec{\beta}_L)\Omega_3\Lambda_3 + 16\frac{\beta_L^2}{\omega^2}$$

$$+ 32\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Lambda_3 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right)^2 - \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega} \right)^2, \quad (8)$$

where $\vec{\alpha}_L = (\alpha_1, \alpha_2, 0)^T$ and $\vec{\beta}_L = (\beta_1, \beta_2, 0)^T$. In telecommunication optical fiber links, optical fibers have only negligible PDL, and PDL exists mainly in some optical components. Then if the fiber section from z to $z + \Delta z$ has no PDL, Eqs. (8) can be simplified as

$$4\beta_B^2\Omega_3\Lambda_3 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right) \cdot \frac{\partial \vec{\Lambda}_B}{\partial z},$$

$$4\beta_B^2(\Omega_3^2 - \Lambda_3^2) + \frac{4\beta_B^2}{\omega^2} = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right)^2 - \left(\frac{\partial \vec{\Lambda}_B}{\partial z} \right)^2. \quad (9)$$

Thus $(\Omega_3^2 - \Lambda_3^2)$ and $\Omega_3\Lambda_3$ can be solved from Eq. (9) once $\vec{\Omega}_B$, $\vec{\Lambda}_B$, and $\vec{\beta}_B$ are measured. In optical fiber links without PDL, the simplified form of Eq. (9) and its experimental verification have been reported [10]. Based on Eqs. (1) and (9), we can calculate $\Omega^2 - \Lambda^2 = (\Omega_L^2 - \Lambda_L^2) + \Omega_3^2 - \Lambda_3^2$ and $\vec{\Omega} \cdot \vec{\Lambda} = \vec{\Omega}_L \cdot \vec{\Lambda}_L + \Omega_3\Lambda_3$. And we already know that DGD and DAS can be expressed as [5]

$$\text{DGD} = \sqrt{\frac{1}{2}[\Omega^2 - \Lambda^2 + \sqrt{(\Omega^2 - \Lambda^2)^2 + 4(\vec{\Omega} \cdot \vec{\Lambda})^2}]},$$

$$\text{DAS} = \vec{\Omega} \cdot \vec{\Lambda} / \text{DGD}. \quad (10)$$

Finally spectrally resolved DGD and DAS can be clearly achieved.

To verify the theoretical findings, an experimental setup is established as shown in Fig. 1. The input polarization state at port 1 of the circulator is tuned by the polarization state generator. The pulse generator generates pulses with an 8 ns pulse width as well as 1 KHz repetition rate and then directly modulates the tunable laser source to emit the optical pulses. The polarization analyzer is composed of a quarter-wave plate followed by a polarizer as well as a 125 MHz photodetector. The fiber under test (FUT) is composed of step-index single-mode fibers (SMFs), dispersion shifted fibers, and dispersion compensation fibers, which are spliced together with a total length of 35 km. Then 1.5 dB PDL is induced by bending the fibers in a small diameter at two positions of the fiber link. A 1 m long SMF is connected with the FUT's far end to introduce Fresnel reflections at its two ends.

To measure $\vec{\Omega}_B$, $\vec{\Lambda}_B$, and $\vec{\beta}_B$, the virtual generalized Mueller matrix method is employed [11]. The whole experiment is divided into two steps. First, the polarization analyzer is connected to port 3 of the circulator. The wavelength is tuned from 1552 to 1557 nm with a 0.2 nm step size. The evolutions of the output polarization states of optical pulses reflected from the two ends of the 1 m SMF with respect to optical wavelength are measured by using the polarization analyzer. Then spectrally resolved DGD and DAS values can be calculated based on the proposed theory. Second, the forward measurement is also performed by using optical pulses in the range 1552–1557 nm with a 0.2 nm step size. The experimental results are shown in Fig. 2. Good agreement between reflectometric and forward measurement results can be observed for both DGD and DAS. This confirms the validity of our proposed reflectometric measurement method. The main measurement error in the experiment is induced by the polarization analyzer. To measure the polarization state of an optical pulse, we need to adjust the quarter-wave plate by hand, which will cause evident measurement error.

In principle, this technique is applicable for both far-end Fresnel reflection and distributed Rayleigh

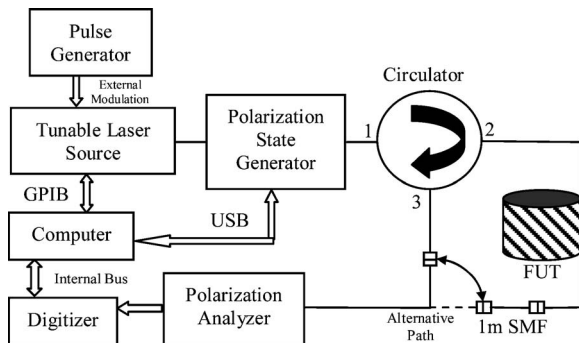


Fig. 1. Experimental setup for spectrally resolved reflectometric measurement of PMD.

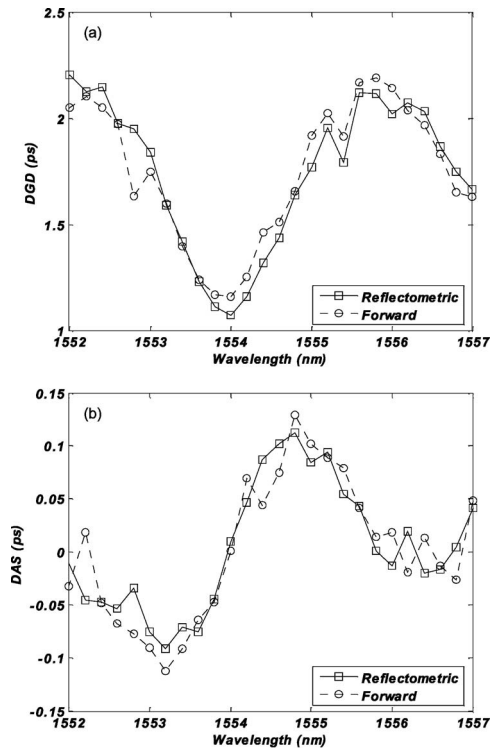


Fig. 2. Comparison of (a) DGD and (b) DAS evolutions in the frequency domain, measured using forward and reflectometric techniques.

backscattering. When the far-end Fresnel reflection is employed, we need to handle the fiber far end as described in the experimental setup. Therefore it is not a truly single-end technique. If the reflected signals are induced by distributed Rayleigh backscattering in optical fibers corresponding to different fiber lengths [2,3], the proposed technique can perform spectrally resolved and spatially resolved PMD measurement in optical fiber links with PDL.

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