

Generalized Mueller matrix method for polarization mode dispersion measurement in a system with polarization-dependent loss or gain

H. Dong, P. Shum, M. Yan, J. Q. Zhou and G. X. Ning

Network Technology Research Centre, Nanyang Technological University, Singapore 637553
hdong@ntu.edu.sg

Y. D. Gong

Institute for InfoComm Research, Singapore 637723

C. Q. Wu

School of Science, Beijing Jiaotong University, China 100044

Abstract: A generalized Mueller matrix method (GMMM) is proposed to measure the polarization mode dispersion (PMD) in an optical fiber system with polarization-dependent loss or gain (PDL/G). This algorithm is based on the polar decomposition of a 4×4 matrix which corresponds to a Lorentz transformation. Compared to the generalized Poincaré sphere method, the GMMM can measure PMD accurately with a relatively larger frequency step, and the obtained PMD data has very low noise level.

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OCIS codes: (060.2310) Fiber optics; (060.2300) Fiber measurement; (060.2400) Fiber properties; (260.5430) Polarization.

References and Links

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1. Introduction

According to the principle state theory, polarization mode dispersion (PMD) is fully described by the principal states of polarization (PSP), differential group delay (DGD) and differential attenuation slope (DAS) between the two PSPs (or equivalently the complex PMD vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$) in an optical fiber system with polarization-dependent loss or gain (PDL/G) [1-3]. To date, many PMD measurement methods have been reported. Among them, Jones matrix eigenanalysis (JME) method is considered to be competent for measuring the PMD in such a system with PDL/G. However, the existing JME method does not present all information of the complex PMD vector [4]. The complex plane method [5] and the generalized Poincaré sphere method [3] need to calculate the derivative of polarization states with respect to optical frequency, which may cause “inaccuracies in the creation and measurement of small differences between optical frequencies and low experimental signal-to-noise ratio in Stokes parameters differences due to polarimeter inaccuracies and other errors” [6]. In an optical fiber system without PDL/G, the Mueller matrix method (MMM) has been demonstrated to be a better technique to attain the low-noise high-resolution PMD data [6]. Very recently, we have derived theoretically the governing equation between the Mueller matrix and the complex PMD vector based on the fact that the Mueller matrix in an optical fiber system with both birefringence and PDL/G satisfies the Lorentz transformation [7]. In this work, we will show that it is possible to find a generalized Mueller matrix method (GMMM) for PMD measurement in an optical fiber system with PDL/G. The paper is organized as follows. In Section 2, we briefly introduce the determination of the Mueller matrix using three input polarization states. In Section 3, we propose a method to extract the complex PMD vector from the measured Mueller matrices based on a polar decomposition technique. Finally in Section 4, experimental results on an optical fiber system with bending-induced PDL confirm the validity and advantages of our GMMM compared with the generalized Poincaré sphere method.

2. Determination of Mueller matrix

When both birefringence and PDL/G exist in an optical system, the 4×4 Mueller matrix \mathbf{M} has been demonstrated to be corresponding to a Lorentz transformation [8], i.e. $\mathbf{M}^T \mathbf{G} \mathbf{M} = \sqrt{\det \mathbf{M} \mathbf{G}}$, where superscript T denotes a transpose and $\mathbf{G} = \text{diag}(1 -1 -1 -1)$ [9]. There are 9 independent bilinear equations relating the elements of the Mueller matrix [9], and it has been shown that only 7 Mueller matrix elements are independent in this case [10]. Therefore, in principle, three non-normalized input polarization states (including the optical power) are sufficient for the determination of Mueller matrix. The output and input Stokes parameters are related by a Mueller matrix as

$$\begin{pmatrix} S_{\text{out}0} \\ S_{\text{out}1} \\ S_{\text{out}2} \\ S_{\text{out}3} \end{pmatrix} = \mathbf{M} \begin{pmatrix} S_{\text{in}0} \\ S_{\text{in}1} \\ S_{\text{in}2} \\ S_{\text{in}3} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} S_{\text{in}0} \\ S_{\text{in}1} \\ S_{\text{in}2} \\ S_{\text{in}3} \end{pmatrix} \quad (1)$$

For ease of calculation, we select the three input polarization states as $(s_{\text{in}0} \ s_{\text{in}0} \ 0 \ 0)^T$, $(s_{\text{in}0} \ -s_{\text{in}0} \ 0 \ 0)^T$ and $(s_{\text{in}0} \ 0 \ s_{\text{in}0} \ 0)^T$. Here $s_{\text{in}0}$ is the input optical power. If their corresponding output polarization states are measured using a polarimeter, then 12 elements in the first three columns of the Mueller matrix can be determined directly. To determine the remaining 4 elements in the fourth column, we consider the following equation (assume Mueller matrix is not singular, which means there is no ideal polarizer in the optical system) [7]

$$\mathbf{M}^{-1} = \frac{1}{\sqrt{\det \mathbf{M}}} \mathbf{G} \mathbf{M}^T \mathbf{G} = \begin{pmatrix} m_{11} - m_{21} - m_{31} - m_{41} \\ -m_{12} & m_{22} & m_{32} & m_{42} \\ -m_{13} & m_{23} & m_{33} & m_{43} \\ -m_{14} & m_{24} & m_{34} & m_{44} \end{pmatrix} / \sqrt{\det \mathbf{M}} \quad (2)$$

On the other hand, \mathbf{M}^{-1} should satisfy another equation as

$$\mathbf{M}^{-1} = \mathbf{M}^* / \det \mathbf{M} = \begin{pmatrix} M_{11} & M_{21} & M_{31} & M_{41} \\ M_{12} & M_{22} & M_{32} & M_{42} \\ M_{13} & M_{23} & M_{33} & M_{43} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{pmatrix} / \det \mathbf{M} \quad (3)$$

where \mathbf{M}^* is the adjugate matrix of \mathbf{M} , M_{ij} is the i, j cofactor of \mathbf{M} . From Eqs. (2) and (3), it's not difficult to get

$$\begin{aligned} m_{14} &= \det \begin{pmatrix} m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}}, & m_{24} &= \det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}} \\ m_{34} &= -\det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}}, & m_{44} &= \det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} / \sqrt{\det \mathbf{M}} \end{aligned} \quad (4)$$

We have known $\sqrt{\det \mathbf{M}} = m_{11}^2 - m_{21}^2 - m_{31}^2 - m_{41}^2$ [9], hence these 4 elements can be calculated according to Eq. (4).

3. Determination of the complex PMD vector, DGD and DAS

If the input polarization state is fixed, the output polarization states at two consecutive optical frequencies can be related by [6]

$$\vec{S}(\omega + \Delta\omega) = \mathbf{M}(\omega + \Delta\omega) \mathbf{M}^{-1}(\omega) \vec{S}(\omega) = \mathbf{M}_\Delta \vec{S}(\omega) \quad (5)$$

where \vec{S} is the 4-dimensional Stokes vector as shown in Eq. (1), $\Delta\omega$ is the step size in optical frequency. Compared to Eq. (1), Eq. (5) means that PMD can be treated as a basic polarization effect in frequency domain. This is equivalent to that birefringence and PDL/G are considered as basic polarization effects in fiber length domain. Because the inverse of a Lorentz transformation and the product of two Lorentz transformations are still Lorentz transformations, from Eq. (5), \mathbf{M}_Δ should also be a Lorentz transformation, and its polar decomposition is [10]

$$\mathbf{M}_\Delta = t_0 \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & \mathbf{m}_R \end{pmatrix} \begin{pmatrix} 1 & \vec{D}^T \\ \vec{D} & \mathbf{m}_D \end{pmatrix} = t_0 \begin{pmatrix} 1 & \vec{D}^T \\ \mathbf{m}_R \vec{D} & \mathbf{m}_R \mathbf{m}_D \end{pmatrix} \quad (6)$$

Here $\vec{0}^T = (0 \ 0 \ 0)$, t_0 is the transmittance for unpolarized light in frequency domain. And

$$\mathbf{m}_R = \begin{pmatrix} \cos \phi + r_1^2 (1 - \cos \phi) & r_1 r_2 (1 - \cos \phi) + r_3 \sin \phi & r_1 r_3 (1 - \cos \phi) - r_2 \sin \phi \\ r_1 r_2 (1 - \cos \phi) - r_3 \sin \phi & \cos \phi + r_2^2 (1 - \cos \phi) & r_2 r_3 (1 - \cos \phi) + r_1 \sin \phi \\ r_1 r_3 (1 - \cos \phi) + r_2 \sin \phi & r_2 r_3 (1 - \cos \phi) - r_1 \sin \phi & \cos \phi + r_3^2 (1 - \cos \phi) \end{pmatrix} \quad (7)$$

$$\mathbf{m}_D = \begin{pmatrix} \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_1^2 & (1-\sqrt{1-D^2})d_1d_2 & (1-\sqrt{1-D^2})d_1d_3 \\ (1-\sqrt{1-D^2})d_1d_2 & \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_2^2 & (1-\sqrt{1-D^2})d_2d_3 \\ (1-\sqrt{1-D^2})d_1d_3 & (1-\sqrt{1-D^2})d_2d_3 & \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_3^2 \end{pmatrix} \quad (8)$$

where $\hat{r} = (r_1 \ r_2 \ r_3)^T$ is a unit vector, ϕ is the rotation angle around \hat{r} in Stokes space. $\vec{D} = D(d_1 \ d_2 \ d_3)^T$ is a vector representing PDL/G in frequency domain. And we know [7]

$$\frac{d\mathbf{M}}{d\omega} \mathbf{M}^{-1} = \lim_{\Delta\omega \rightarrow 0} \frac{\mathbf{M}_{\Delta} - \mathbf{I}}{\Delta\omega} = \begin{pmatrix} \eta_\omega & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1 & \eta_\omega & -\Omega_3 & \Omega_2 \\ \Lambda_2 & \Omega_3 & \eta_\omega & -\Omega_1 \\ \Lambda_3 & -\Omega_2 & \Omega_1 & \eta_\omega \end{pmatrix} \quad (9)$$

Here \mathbf{I} is an identity matrix, $\eta_\omega = \frac{d \ln^4 \sqrt{\det \mathbf{M}}}{d\omega}$. $\vec{\Omega} = (\Omega_1 \ \Omega_2 \ \Omega_3)^T$ and $\vec{\Lambda} = (\Lambda_1 \ \Lambda_2 \ \Lambda_3)^T$ are the real and imaginary parts of the complex PMD vector \vec{W} . Comparing Eq. (6) and Eq. (9), assuming the higher-order PMD effects between the two frequencies are negligible, we have

$$\vec{\Lambda} = t_0 \vec{D} / \Delta\omega, \quad \vec{\Omega} = \phi \hat{r} / \Delta\omega \quad (10)$$

Here ϕ and \hat{r} can be extracted from \mathbf{m}_r based on same equations as the Eqs. (6), (7) and (8) in Ref [6]. To avoid measurement ambiguities [4], the condition $|\vec{\Omega}| \Delta\omega < \pi$ should be satisfied. Finally, DGD and DAS can be calculated based on the following equation [3]

$$DGD = \text{Re}(\sqrt{\vec{W} \cdot \vec{W}}), \quad DAS = \text{Im}(\sqrt{\vec{W} \cdot \vec{W}}) \quad (11)$$

4. Experimental setup and results

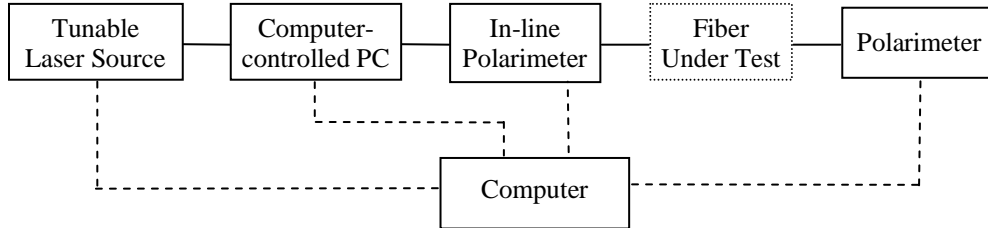
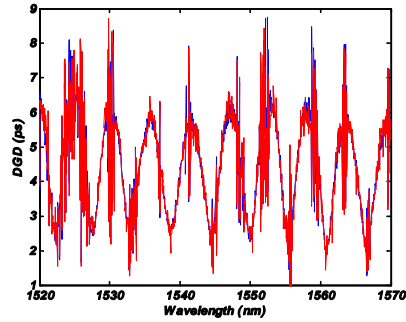


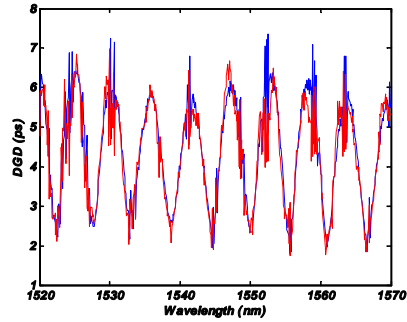
Fig. 1. Experimental configuration for PMD measurement in an optical fiber system.

To verify the proposed theory and measurement method, an experimental setup is established as shown in Fig. 1. The fiber system under test (FUT) comprises three Hi-Bi fiber sections (PANDA type, with 3m-long length and 4ps DGD for each) interleaved by two single-mode fiber sections (1m-long for each). In particular, the Hi-Bi fiber in the middle is spooled on a drum with a 1.5cm diameter, so bending-induced PDL exists in this system [11]. The tunable laser source emits completely polarized light with a stable polarization state and a 700kHz linewidth. The computer-controlled polarization controller (PC) as well as the in-line polarimeter are used to generate and monitor the appropriate input Stokes parameters. The polarimeter in the end is used to measure the output Stokes parameters. All components except FUT are controlled by a computer through GPIB port, USB port and I/O card.

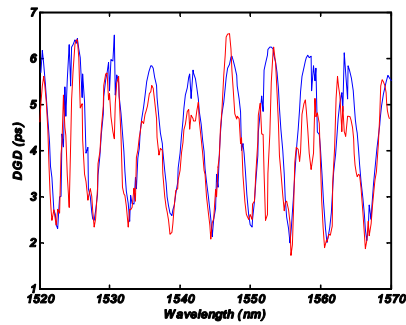
In our experiments, three input polarization states are generated in turn as $(s_{in0} \ s_{in0} \ 0 \ 0)^T$, $(s_{in0} \ -s_{in0} \ 0 \ 0)^T$ and $(s_{in0} \ 0 \ s_{in0} \ 0)^T$. The wavelength range is from 1520nm to 1570nm. Based on the same measured data, we calculate the DGD and DAS using both GMMM and the generalized Poincaré sphere method. The obtained results are compared in Fig. 2. The wavelength step sizes are chosen as 0.05nm, 0.1nm, 0.2nm and 0.5nm in different measurements to verify which PMD measurement algorithm tolerates larger frequency step size for attaining low-noise PMD data.



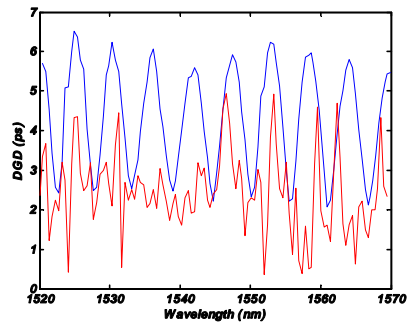
(a) DGD $\Delta\lambda=0.05\text{nm}$



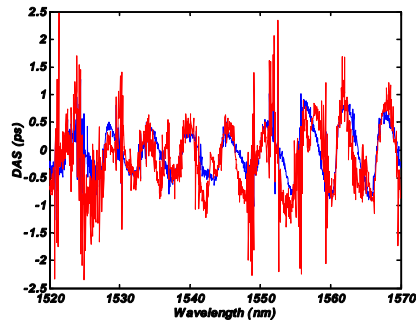
(b) DGD $\Delta\lambda=0.1\text{nm}$



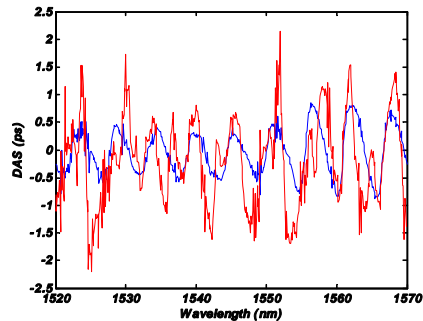
(c) DGD $\Delta\lambda=0.2\text{nm}$



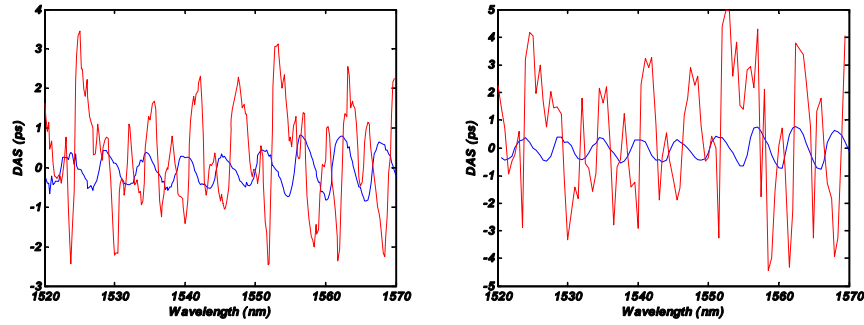
(d) DGD $\Delta\lambda=0.5\text{nm}$



(e) DAS $\Delta\lambda=0.05\text{nm}$



(f) DAS $\Delta\lambda=0.1\text{nm}$



(g) DAS $\Delta\lambda=0.2\text{nm}$ (h) DAS $\Delta\lambda=0.5\text{nm}$
 Fig. 2. DGD (a-d) and DAS (e-h) measurement results calculated by GMM (blue lines) and the generalized Poincaré sphere method (red lines) with different wavelength step sizes.

From Fig. 2, we clearly know that GMM and the generalized Poincaré sphere method give close results of DGD and DAS and exhibit large noise when the wavelength step size is less than 0.1nm. But when the wavelength step size is larger, GMM can well maintain its measurement accuracy, and with lower noise; the generalized Poincaré sphere method, however, gives incorrect results due to its using differentiation.

We point out that the PDL can also be obtained from measured Mueller matrices based on the following equation [12]

$$PDL = 10 \log \frac{m_{11} + \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}}{m_{11} - \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}} \quad (12)$$

The measured relationship between PDL and wavelength in FUT is plotted in Fig. 3.

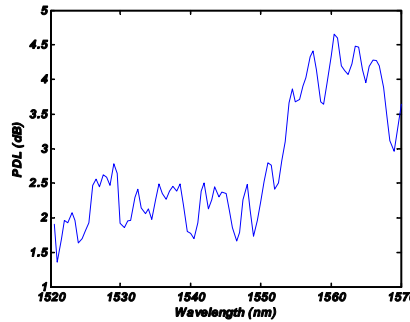


Fig. 3. Measured PDL at different wavelength in the fiber system under test.

5. Conclusion

We presented a generalized Mueller matrix method for PMD measurement in an optical fiber system with PDL/G based on the polar decomposition of a 4×4 matrix which satisfies the Lorentz transformation. Compared to the generalized Poincaré sphere method, the GMM allows a larger frequency step size in measurements, and its results appear to be accurate with high signal-to-noise ratio.

Acknowledgments

This work is partially supported by the project 042 101 0015 of the Agency for Science, Technology and Research (A*Star), Singapore and the Open Fund of Key Laboratory of Optical Communication and Lightwave Technologies, Beijing University of Posts and Telecommunications, Ministry of Education, P.R.China.