

Coupling coefficient of two-core microstructured optical fiber

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Abstract

In this paper, coupled-mode theory is applied to a two-core microstructured optical fiber for the first time to calculate the coupling coefficients for different fiber structures by employing a simple effective index model approach. The dependence of the mode coupling properties upon the geometrical parameters of the two-core structures (air-hole arrangement, hole size, and pitch size) and wavelength are evaluated systematically. The effective index coupled-mode theory is compared with the finite-element method based super-mode theory in details and the results show good agreement. The coupling characteristics are proven to be insensitive to the longitudinal strain by considering the photoelastic effects.

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1. Introduction

Microstructured optical fiber (MOF) is a new class of optical fiber that has emerged in recent years [1]. It is formed by an array of air holes running along the fiber length and guides light by total internal reflection between the solid core and the holey cladding region. This new type of fiber is fascinating because of its various novel properties, including endlessly single-mode operation [2], scalable dispersion and nonlinearity, and the surprising phenomenon of a short wavelength bend loss edge [3], etc. These novel transmission characteristics make such fiber ideal candidate for wide range of applications in optical communication systems. In particular, by considering the MOF's unusual cladding structure, researchers have fabricated many novel devices based on mode coupling effects, such as long period gratings (LPG) [4], fiber Bragg gratings (FBG) [5], fused couplers [6], multi-core fibers [7], etc. Among these devices, multiple-core MOFs can be easily

obtained by simply using multiple solid rods as defects in the stack and drawing fabrication process. Many useful devices based on a symmetric two-core fiber can be realized, such as directional couplers, wavelength division multiplexers, and polarization beam splitters. In conventional fibers, coupled-mode theory has been systematically analyzed and mode coupling in conventional two-core structures has been studied in details [8]. For two-core MOF, many works have been done on the numerical calculation of coupling lengths [12,13]. However, these numerical methods are very computationally inefficient and so far the coupled-mode theory in two-core MOFs has not been investigated. Hence, in this paper we build up the coupled-mode theory for MOFs based on an effective index model. The dependence of coupling coefficient on the microstructure of the cross-section, such as the air-hole arrangement, hole-to-hole distance and air-hole size will be analyzed in detail. The wavelength dependence of the coupling properties is also investigated. The environment, such as temperature, stress, compression and strain, can change the microstructure. Due to the photoelastic effects, mechanical strain in the longitudinal direction will not only

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reduce the scale of the fiber cross-section, but also cause the stress re-distribution inside the fiber material. For this reason, we characterize, for the first time to our knowledge, the variation of the coupling properties in this novel two-core structure under different strain conditions.

2. Effective index coupled-mode theory

In a standard optical fiber, light launched into the core at angles greater than the critical angle can propagate for long distances in the high refractive index interior with little loss in intensity by means of total internal reflection. As is well known, there is an exponentially attenuated component of the internally reflected wave in the transverse direction that extends into the low-index region. If another high refractive index region is placed near enough, the attenuated wave can couple into this region and propagate within it. Thus, energy transfer can occur between the two waveguides. In conventional two-core fiber, energy transfer occurs in a spatially periodic fashion between the two cores, and this phenomenon can be modeled accurately with the coupled-mode theory to study devices based on evanescent-field coupling.

In case of the two-core MOF structure as shown in Fig. 1(a), where Λ is the hole-to-hole lattice distance, d is the air-hole diameter, S is the separation between the centers of two cores and the refractive index of background material (fused silica) is taken to be 1.45. Since the coupled-mode theory in a conventional two-core fiber has been systematically investigated, we use the effective index method to get the equivalent step index profile of the two-core MOF. The fundamental space-filling mode [9] in the infinite periodic cladding is calculated firstly and the whole periodic space is divided into hexagonal unit cells. The equivalent structure is obtained as shown in Fig. 1(b), where the effective index of the cladding acts as refractive index of the homogenous cladding material and pure silica acts as core. R is the equivalent core radius and strongly depends on the microstructure. We calculated the fundamental mode of two-core MOF and the equivalent two-core fiber, and the same effective mode index for both structures is achieved by tuning the equivalent core

radius R between $\sqrt{3}\Lambda/2$ and Λ . An analytical expression of the effective core radius has been shown recently [10]

$$R = c_1 \Lambda / \left\{ 1 + \exp \left[\left(\frac{d}{\Lambda} - c_3 \right) / c_2 \right] \right\}, \quad (1)$$

where c_1 , c_2 and c_3 are 0.686064, 0.265366, and 1.291080, respectively. This effective index method is used here to build up a simple and straightforward coupled-mode theory, and we call it effective index coupled-mode theory. For this paper, we used a very simple structure to demonstrate the coupled-mode theory in a microstructure: the two separate cores are identical, lossless and monomode. The generalized analysis for non-identical, multi-mode cores will be similar to conventional structures with homogeneous cladding. The detailed analysis has been presented in [8].

The coupling coefficient C_{pq}^{js} is a quality factor of the overlap of the p th mode in core j ($e_p^{(j)}$) and the q th mode in core s ($e_q^{(s)}$). Its general form is

$$C_{pq}^{js} = \frac{\omega}{2} \int_{A^{(s)}} (\epsilon^{(s)} - \epsilon) e_p^{(j)} \cdot e_q^{(s)} dA, \quad (2)$$

where both fields are at frequency ω , $(\epsilon^{(s)} - \epsilon)$ is the difference between the dielectric constants of the core s and its cladding, and $e_p^{(j)}(x, y)$ is the transverse electric field profile of mode p in core j in the absence of core s . $e_q^{(s)}(x, y)$ is the transverse electric field profile of mode q in core s in the absence of core j . For single-mode two-core fibers in which the two cores are identical, the coupling coefficients between the two cores are equal, i.e., $C_{12} = C_{21}$. Then Eq. (2) can be simplified as [11]

$$C_{12} = C_{21} = C = \sqrt{2\Lambda} \frac{u^2}{RV^3} \frac{K_0(wS/R)}{K_1^2(w)}, \quad (3)$$

where S is the separation between the two cores, R is the radius of the cores, V is the normalized frequency which is given by: $V = k_0 R n_{co} \sqrt{2\Lambda}$, where $\Delta = (n_{co}^2 - n_{cl}^2)/2n_{co}^2$ is the refractive index difference between core and cladding with n_{co} and n_{cl} being the refractive index of the core and the cladding. The modal parameter u is a solution of the characteristic equation

$$uJ_1(u)/J_0(u) = wK_1(w)/K_0(w), \quad (4)$$

where $w = \sqrt{(k_0 R)^2 (n_{co}^2 - n_{cl}^2) - u^2}$, $k = 2\pi/\lambda$, J_0 and J_1 are the zeroth and first order Bessel functions of the first kind, and K_0 and K_1 are the zeroth and first order modified Bessel functions of the second kind, respectively.

Eqs. (3) and (4) cannot be solved analytically, so we need to use numerical methods to calculate the coupling coefficient. For the equivalent two-core structure of MOF, core-to-core distance S , effective core radius R and effective cladding index n_{cl} are the three principle parameters to control the coupling coefficient. For MOFs, decreasing the air-hole size while keeping the pitch size (Λ) constant is equivalent to decreasing core cladding index contrast since cladding air-filling fraction is smaller. This increases the coupling coefficient significantly as shown in

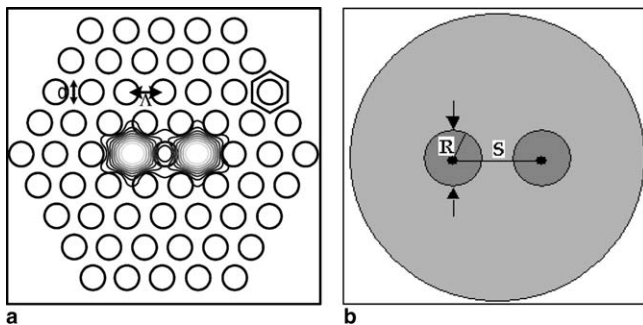


Fig. 1. (a) Cross-section of a two-core MOF formed with hexagonal unit cells, the transverse mode profile shown with the contour map; (b) cross-section of the equivalent two-core fiber.

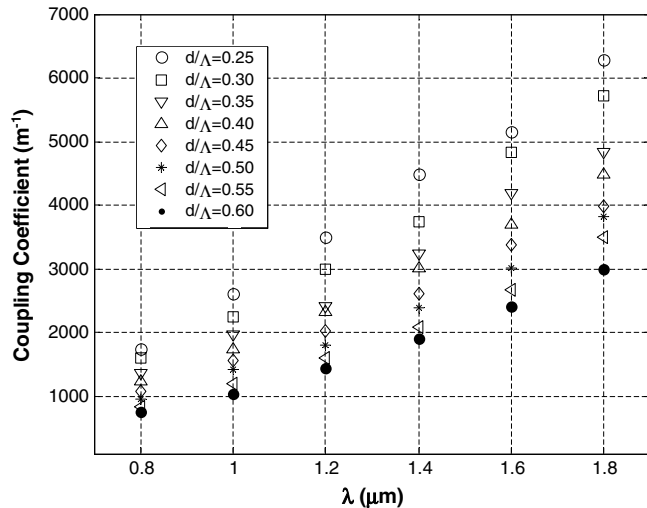


Fig. 2. Coupling coefficient for structure shown in Fig. 1(a) at different wavelength, $\Lambda = 4 \mu\text{m}$. From top to bottom, $d/\Lambda = 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6$.

Fig. 2. If the air-hole size is fixed, increase of the pitch size will cause the increase of core size, the core-to-core distance and the effective cladding index. Thus, the coupling coefficient will increase as well. The coupling coefficient in a conventional two-core fiber should be strongly wavelength-dependent as calculated in [8], however, spectral response in the case of two-core MOF shows a less wavelength dependence than normal two-core fiber. This can be explained from Eq. (2): dielectric constant of the cladding in MOF decreases when we increase wavelength, this effect eliminates certain dependence of C on ω .

Arranging the position of the solid rods in the stacking process can easily change the separation between the two cores. Two-core MOF structures with different core-to-core distance are shown in Fig. 3, where $S = 2\Lambda, 3\Lambda, 4\Lambda$. Calculated results are shown in Fig. 4 from top to bottom, respectively. The coupling coefficient is decreased by about 10 times when increasing the S by one lattice distance, while keeping the same air-filling fraction by fixing the d/Λ ratio.

3. Comparison and discussion

The accuracy of the coupling coefficient depends on the modal parameters of the fiber, u and w from the characteristic Eq. (4). In case of MOF, the two parameters accuracy depends on the accurate calculation of core index n_{co} , cladding index n_{cl} and the effective core radius R . However, n_{co} is fixed as the fused silica index 1.45. R depends on the air-filling fraction of the microstructure according to Eq. (1), which is equivalent to that R strongly depends on n_{cl} . So we can conclude that the accuracy of coupling coefficient depends on the accurate calculation of effective cladding index. The sensitivity of coupling coefficient to cladding index is shown to be around $3.55 \text{ m}^{-1}/0.1\%$, as shown in Fig. 5(a).

Among various numerical modeling, full-vectorial finite-element method (FEM) is proven to be the most accurate method. However, for the complex holey structure in MOF, the number of mesh will increase dramatically and the computational efficiency is rather low [12,13]. Here, we employ the effective index method, which is proven to be the simplest and most efficient. The unit cell of the periodic microstructure in the cladding is treated as a single waveguide. The effective modal index of the unit cell is to be obtained as the effective cladding index value of the corresponding MOF. Then we compare with the results with FEM. Fig. 5(b) shows that the discrepancy of the effective cladding index between two methods is rather small for a large wavelength range. As is well known, the accuracy will increase even more when we increase the air-filling fraction because the modes will be tightly confined in the core. To further demonstrate the validity of this effective index model, we calculate the coupling coefficient based on Eq. (3), and compare it with the published results. In [13], coupling length L_c rather than C was calculated based on FEM. The coupling length L_c is related to the coupling coefficient C by the equation $L_c = 2\pi/C$. The coupling lengths calculated from both methods are compared in Fig. 5(c). When the fiber core size is smaller than wavelength, the coupled-mode theory will not be valid any more. Thus, we observed a relatively large discrepancy at small Λ . However, the discrepancy is small for most

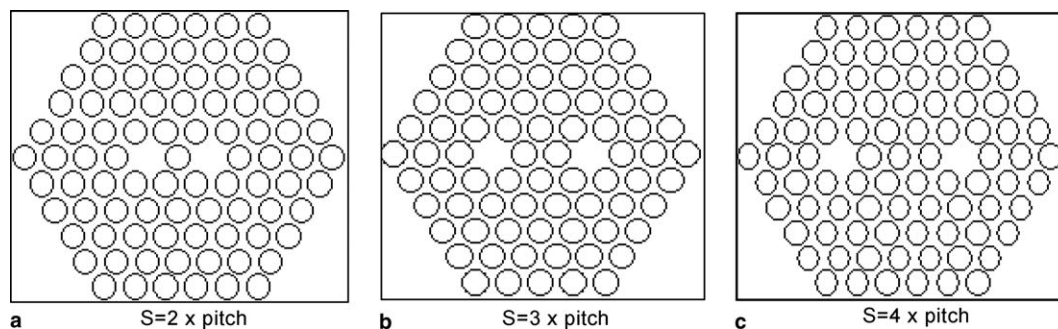


Fig. 3. Two-core MOFs with three different core separations: (a) $S = 2\Lambda$, (b) $S = 3\Lambda$, (c) $S = 4\Lambda$.

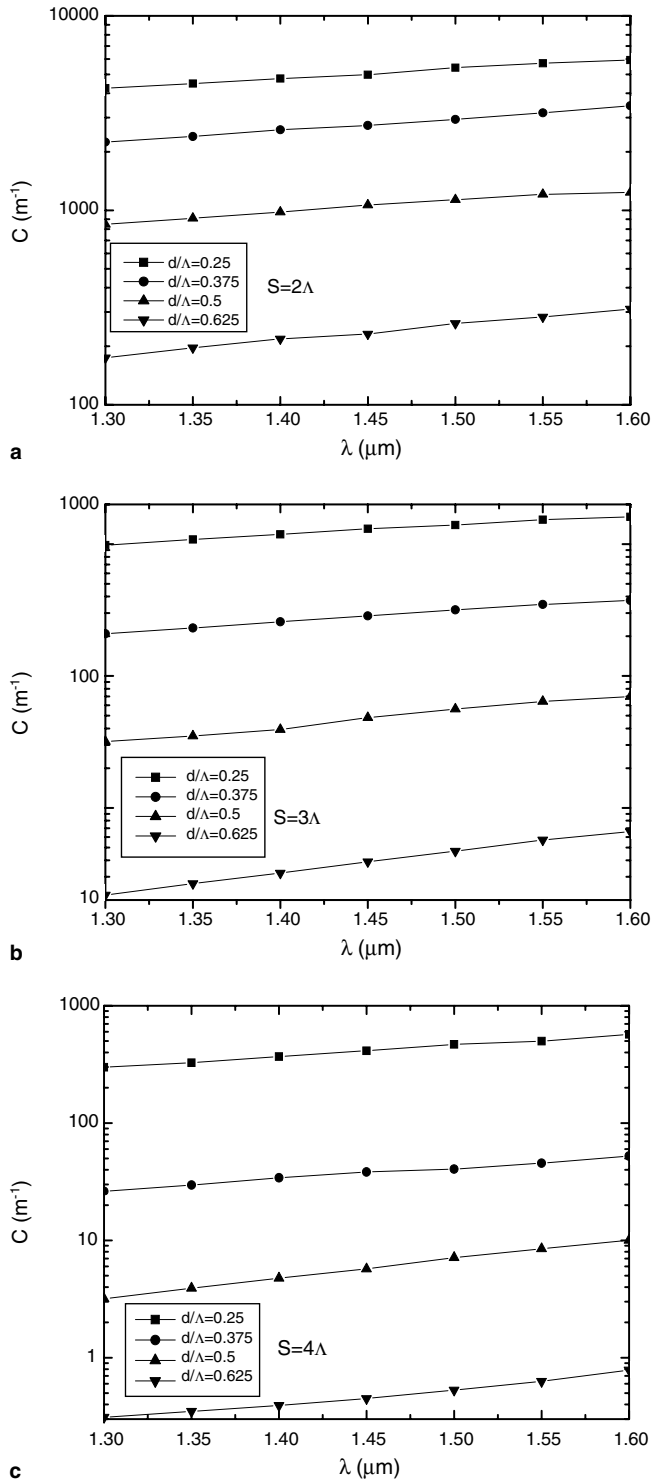


Fig. 4. Coupling coefficients of the three structures in Fig. 3: $\Lambda = 4 \mu\text{m}$, $d = 1.0, 1.5, 2.0, 2.5 \mu\text{m}$.

structures. The difference in L_c values calculated by FEM and effective index model is diminishing when the scaling parameter Λ is increased. Therefore, the established effective index coupled-mode theory is sufficiently accurate for illustrating the mode coupling properties of the most structures, and it improves the efficiency dramatically.

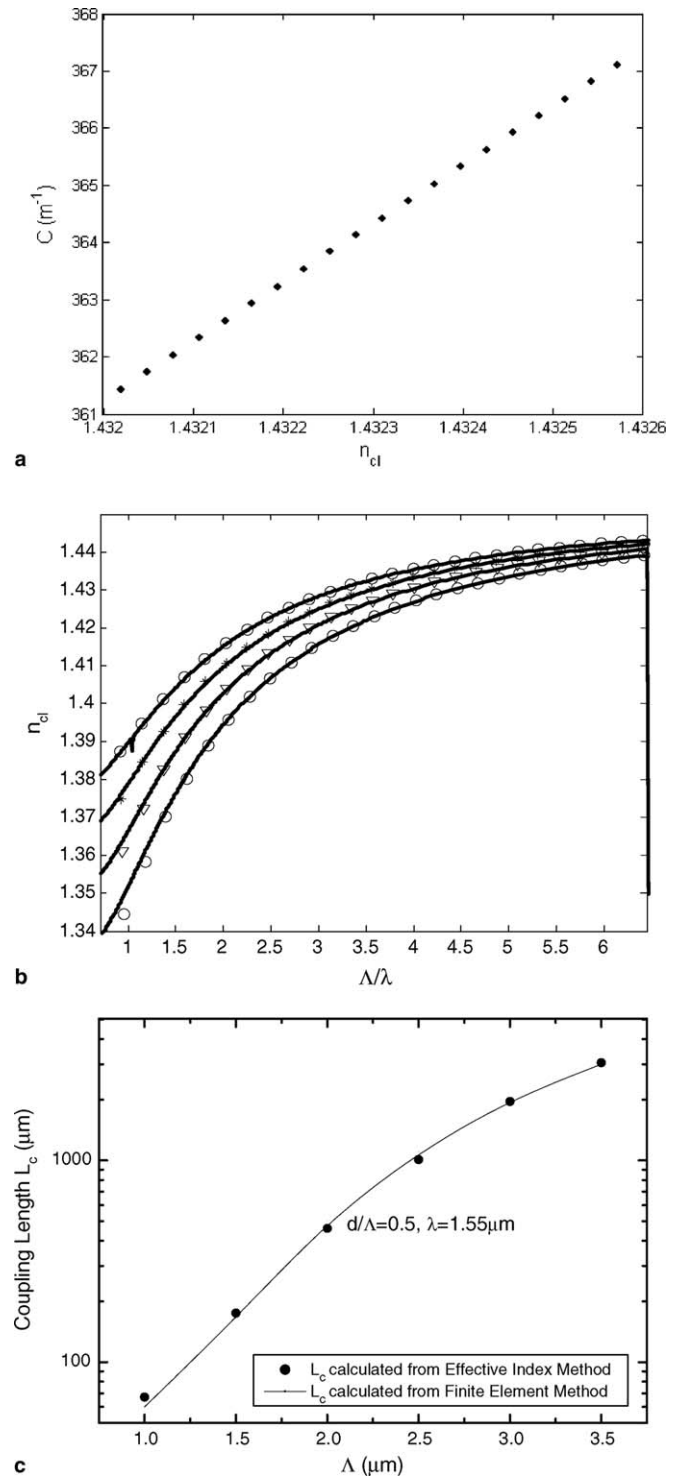


Fig. 5. (a) Sensitivity of coupling coefficient to the cladding index. (b) Comparison of effective cladding index calculated by effective index method (solid line) and finite-element method (shaped mark), from top to bottom, $d/\Lambda = 0.40, 0.45, 0.50, 0.55$. (c) Comparison of coupling length calculated by effective index method and finite-element method to test the validity of the effective index coupled-mode theory.

In the FEM based calculation, the two cores and the surrounding media are combined as a single waveguide, so it is a super-mode theory. It is well known that single-

circular-core fiber has degenerate HE_{11}^x and HE_{11}^y modes propagating with the same phase velocity. When we have two cores, the infinite rotational symmetry of the twin-core is broken. The x-polarized fundamental mode (correspond-

ing to HE_{11}^x mode in a single-circular-core fiber) breaks into two modes: even HE_{11}^x and odd HE_{11}^x (as shown in Fig. 6). Notice they have different propagation constants, meaning the two modes propagate out of phase most of the time. But when at the exact moment, i.e., two modes have zero (or $2N\pi$) phase difference, left core has almost zero energy due to destructive interference. Similarly if two modes are in anti-phase, the right core has almost zero energy. The effective indexes of four different modes are shown in Fig. 6(c). It is obvious that the coupling coefficient is thus slightly different for the x-polarized and y-polarized modes. So polarization effects are negligible in the effective index couple-mode theory for simplicity.

4. Strain sensitivity test

By evaluating the coupling characteristics in two-core MOF, we find the dependence on pitch and core diameter. These geometrical parameters can be changed by environment, such as temperature, strain or stress. For example, when stretching the fiber in the longitudinal direction, the fiber is elongated, the air-hole size and the pitch size are decreased. We can divide the whole microstructured cross-section into a number of hexagonal unit cells as shown in Fig. 1(a): A_{hex} is the area of the unit hexagonal; A_{cir} is the area of the circular air hole; L is the fiber length. The air-filling fraction can be calculated approximately from solid silica contributing to the total mass of the fiber. By mass conservation, we can get: $V = (A_{\text{hex}0} - A_{\text{cir}0})L_0 = (A_{\text{hex}} - A_{\text{cir}})L$. We assume the air-hole diameter to pitch ratio is constant since there is no temperature variation. Then $(A_{\text{hex}} - A_{\text{cir}})$ is proportional to A^2 , and $A_0^2/A^2 = L/L_0$. The change of fiber length will also affect the strain-optic induced refractive index. This photoelastic effect also needs to be considered during the modeling, which accounts for the change of the refractive index due to the strain [14,15]. Thus, we deduced the relationship of structure change and index change under strain:

$$\frac{\Delta A}{A} = \frac{\Delta L}{2L_0} = \frac{(1-2\nu)F}{2EA}, \quad (5a)$$

$$\frac{\Delta n}{n} = \frac{n^2 F}{2EA} (1-2\nu)(2\rho_{12} + \rho_{11}) = n^2(2\rho_{12} + \rho_{11}) \frac{\Delta L}{2L_0}, \quad (5b)$$

where ν is the Poisson's ratio, F is the applied strain force on the fiber, E is the Young's modulus for silica, A is the fiber cross-section, and ρ_{11} and ρ_{12} are the components for the strain-optic tensor.

This strain elongation causes the decrease of core size R as well as the core-to-core separation S , and these two parameters cancel the influence of each other to the coupling coefficient; longitudinal strain causes the re-distribution of the stress inside the glass material, which increases the refractive index according to the photo elastic relationship. For structures with small air-filling fractions, the index variation caused by strain in the core and cladding regions is roughly equivalent to each other. When the $d/$

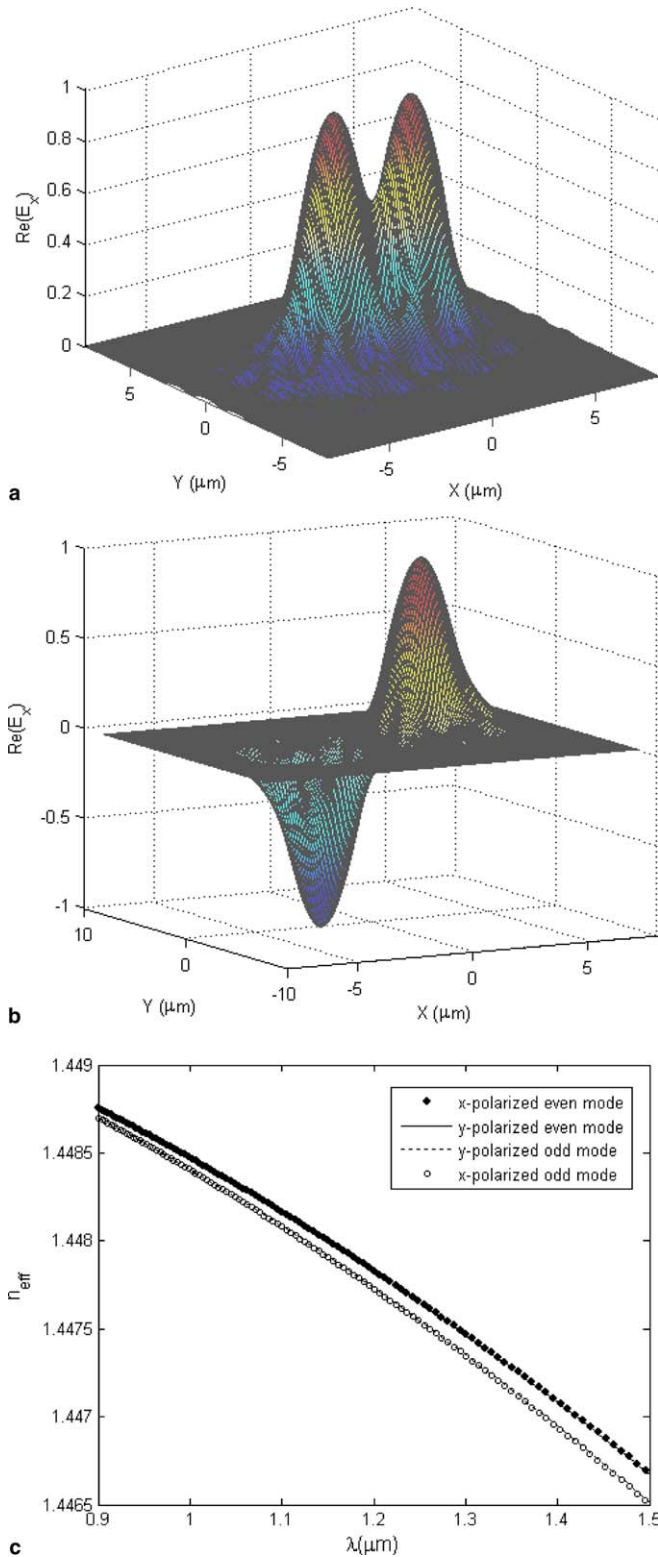


Fig. 6. (a) x-polarized even mode; (b) x-polarized odd mode; (c) effective mode index of four super-modes.

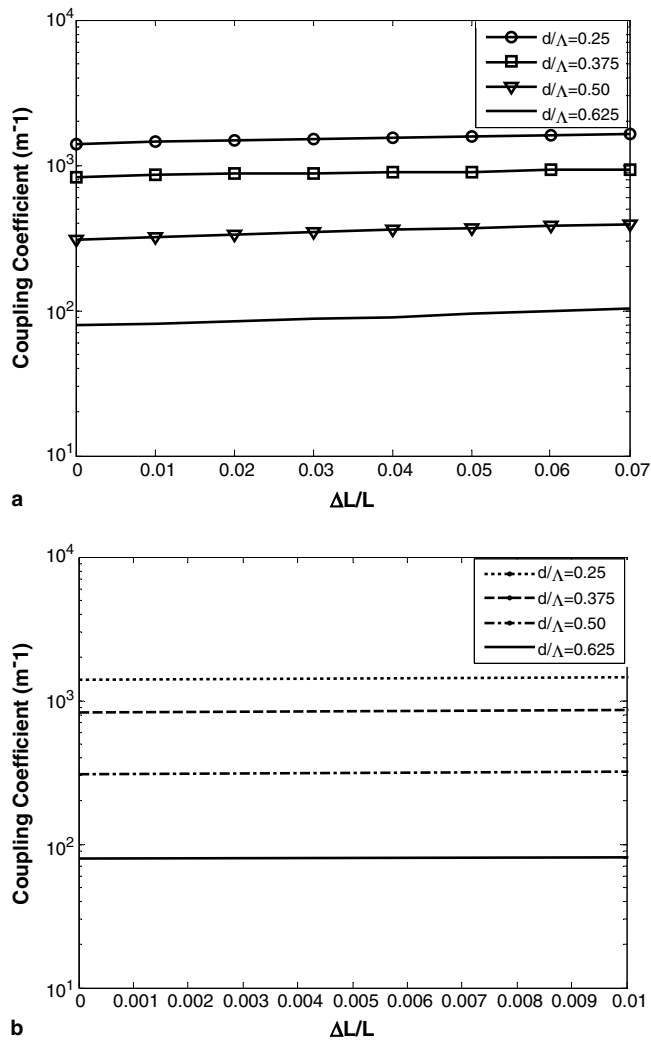


Fig. 7. Coupling coefficients of two-core MOF (a) with different fiber elongation length and different air-filling fractions: $\Lambda = 4 \mu\text{m}$, $\lambda = 1.55 \mu\text{m}$; (b) within the strain limit.

d/Λ ratio increases, index variation cannot balance each other for the core and cladding, thus the photo elastic influence to the variation of coupling becomes significant. The slope of the coupling coefficient curve vs. strain is shown in Fig. 7(a) to get larger in the two-core MOF with increased d/Λ . Since the breaking strain of silica MOFs is measured to be on the order of 1%, we consider the geometric parameter and index variations for two-core MOF under different strain conditions below the breaking threshold. We calculated the coupling coefficients, and investigated the elongation dependence of coupling characteristics, where $\lambda = 1.55 \mu\text{m}$, $\Lambda = 4 \mu\text{m}$, $d/\Lambda = 0.25, 0.375, 0.5, 0.625$. The ultra-flattened coupling coefficient spectrum with fiber elongation $\Delta L/L$ is shown in Fig. 7(b), which shows good performance in terms of strain insensitivity ($0.6 \times 10^{-4}/\text{m} \mu\epsilon$).

5. Conclusion

In conclusion, we have theoretically analyzed the coupled-mode theory in two-core MOF using a simple and efficient effective index model. The coupling characteristics of two-core MOF were numerically investigated for the first time in these symmetric structures. Coupling coefficient will increase when we increase the lattice pitch or decrease the air-hole size. And we found that the wavelength dependence of the coupling in MOF will be smaller than that in conventional two-core structures. Effective index based coupled-mode theory is compared with the FEM based super-mode theory, and the results show good agreement. Good strain insensitivity of the coupling coefficient $0.6 \times 10^{-4}/\text{m} \mu\epsilon$ was observed, making them potential candidates for applications in harsh environments, such as space aircraft, nuclear power plants, and the chemical industry. As MOFs are revolutionizing the physics of optical fiber waveguiding, two-core and even multi-core coupling structures will lead to more applications for optical communication.

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